TRIUMF Summer Institute, July-August 2009, Statistics : Thomas Junk

Homework Assignment, Day 2

1) A search for a new particle is done by counting events passing a selection and comparing them against the prediction of the Standard Model, and also that including a new particle.

Let's say that the SM is the null hypothesis H_0 and it predicts 1.2 events, called the background rate b. Let's assume for the moment that there is no uncertainty on b. Suppose that a model of new physics predicts a signal rate s that adds incoherently to b so that the prediction in the test hypothesis H_1 is $s + b = 5.3$ events, also with no systematic uncertainty. Suppose that when the experiment is run, one event is observed.

a) What is the p-value for testing the null hypothesis H_0?

b) What is the p-value for testing the test hypothesis H_1?

c) Calculate $\mathrm{CL_s}$.

d) How small does b have to be in order for a single event to constitute a discovery at 5σ (p-value=2.87×10^{-7})?

e) What is the median expected p-value for testing H_0 assuming that H_1 is true?

2) Now suppose that the roles are reversed. A new physics process affects the predicted yield by *reducing* it. This can happen most easily in cases in which the existence of a new particle opens up a decay mode of a known particle (one which is heavy enough to decay into particles not yet discovered), where the new decay is not easily detected (say the new particle is invisible), or simply fails to pass the selection requirements at the rate of the standard decays.

Take the numbers from before, but exchange H_0 and H_1. The SM (H_0) predicts 5.3 events, the exotic model (H_1) predicts 1.2 events, and one event is observed.

a) What is the p-value for testing the null hypothesis H_0?

b) What is the p-value for testing the test hypothesis H_1?

c) Calculate $\mathrm{CL_s}$.

d) What is the median expected p-value for testing H_0 assuming that H_1 is true?

3) Explain why the Feldman-Cousins prescription never gives empty intervals.